Private Money as a Competing Medium of Exchange

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Abstract

By introducing costly centralized exchange into a traditional search theoretic decentralized market, where money can mediate trade, we show private money can arise endogenously because it is profitable for the centralized intermediary to satisfy the demand for money that will naturally arise. Trading money to another agent for a good, rather than trading good for good through the centralized intermediary, allows the money holder to obtain the good while avoiding the fee the centralized intermediary must charge to cover its costs. The model provides a set of pure strategy monetary equilibria where the quantity of privately issued money is a monotonically increasing function of the confidence agents have in money. The model indicates that the circulation of fiat money is explained by the role it can play in reducing transaction costs more so than the role it can play as a medium of exchange in resolving the double coincidence of wants problem.
1 Introduction

Starr (2003, p. 456) has suggested that the focus on the absence of double coincidence of wants as an explanation for the monetization of trade— as distinct from transaction costs— may miss a significant part of the underlying causal mechanism. He presents a general equilibrium model, where there is no double coincidence of wants problem, and shows that a form of commodity money arises when transactions costs motivate traders to trade through one particular good as they seek other goods. Here, we present a model that includes both a decentralized market, where there is a double coincidence of wants problem, and a centralized market that can be operated if transactions costs are incurred. This allows us to relate money’s ability to resolve the double coincidence problem to its ability to reduce transactions costs.

The intuition behind this result is the idea that, if competing mediums of exchange coexist, each must have an advantage and a disadvantage. The centralized intermediary in our economy is a Walrasian clearing house, which we call a store. The advantage of the store is it is a completely effective medium of exchange. With the double coincidence of wants problem resolved by the store, money must solve a different problem if it is to circulate. The store’s disadvantage is it must effectively charge traders a transactions cost to cover its costs. This is what makes it possible for money to compete as an alternative medium of exchange.

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1Starr’s model captures Tobin’s (1980) suggestion that, “The use of a particular language or a particular money by one individual increases its value to other actual or potential users.” Starr further uses his model to explain the circulation of fiat money by imposing the assumption that only the government sponsored money can be used to pay taxes. When trading volume reduces the average transactions cost, the volume of trade necessitated by the payment of taxes can give the government issued, intrinsically useless asset a low enough transactions cost that it becomes the economy’s money.

2We use the term store for our mediator primarily to avoid confusing our mediation conception with other conceptions in the literature, including trading post and middleman.

3It is worth noting that agents hold money with the hope avoiding the store’s transactions cost, the motive for holding money is not a pure transactions motive, but it has a "speculative" (in the Keynesian sense) component to it. In our model demand for money depends on the transaction cost in the centralized market. As the transaction cost goes up the opportunity cost of holding money (i.e. postponing consumption) goes down, so the demand for money goes up (this is similar in spirit to Keynesian speculative demand for money where demand for money is inversely related to interest rate).
This work is most closely related to that of Camera (2000). He similarly introduces a
centralized market, which is costly to operate, that can compete with money circulating in
a decentralized search sector. Camera and Li (2008) show that money can co-exist with
credit where record keeping is limited and imperfect. In both papers the primary question
is whether money is still essential, and they find that it is, under a rather robust set of
conditions. Our model is different in two important respects.

First, in our model money is not introduced by government. Rather, money is introduced
by one of the agents (store) of our model. In certain disequilibrium states, the store is
motivated to create and introduce new additional money because it can earn a windfall profit
by doing so, and it must do so to prevent another store from capturing the centralized market.
Thus, the economy’s money is “private”, being introduced by the centralized intermediary. A
demand for money arises when traders expect as much or more utility from trading through
money than from trading through the store. Our monetary equilibrium existence theorem
identifies the environmental conditions under which money can be profitably introduced and
circulate as a medium of exchange in competition with centralized intermediation. In our
model the equilibrium quantity of money in the economy is determined by the economic
fundamentals.

Second, agents in our model do not choose between trading in the centralized and de-
centralized sectors as in the Camera (2000) model. Rather, they always participate in the
decentralized sector first and have the store as a backup option. Intuitively, one can think
that an agent holding money is walking to the store, through the decentralized trading en-
vironment, hoping that the more attractive money trade can be made rather than the less
attractive store trade. When agents can trade good for good through a centralized interme-
diary (i.e., store), they will only trade good for money when they speculate that the terms
of trade in next period’s decentralized market are attractive enough to compensate for the
risk of not making a trade plus the cost of waiting to consume.

Shi (2006, p. 644), referring to Wallace (1980), defines fiat money as a circulating object
that is (a) intrinsically useless and (b) not backed by government policy. The money in our
model is fiat money by this definition. The circulating money medium is an intrinsically useless IOU issued by the store. While the store “backs” the money by agreeing to exchange good for money whenever a trader so desires, no trader will ever accept money expecting to return it to the store, for doing so is not rational. Rather, money is only held with the expectation of trading it to another agent for good in the decentralized market.

As in the typical fiat money model, if agents do not have enough confidence that others will accept money, no monetary equilibrium will arise because holding money is too risky. What distinguishes our model is what happens when the threshold level of confidence is reached. In the typical model, the confidence agents have in money must be consistent with the amount of money issued by government. However, because money is issued privately in our model, according to the profitability of doing so, the quantity of private money adjusts to the confidence agents have in money. The quantity of money circulating in equilibrium is higher when confidence in money is higher. To our knowledge, our model is unique in providing a set of pure strategy equilibria where the economy’s equilibrium money supply is a monotonically increasing function of the confidence agents have in money.

Money in our economy may or may not be essential\textsuperscript{4}. Consistent with Kiyotaki and Wright (1993), when barter is easy enough, or when agents don’t discount the future enough, more money only reduces welfare. Alternatively, when barter is difficult enough or agents discount the future enough, the effect of money on welfare depends upon the operating cost incurred by the store. At a low store operating cost, more money only increases welfare. At a high store operating cost, more money only decreases welfare. At intermediate store cost levels, there is a welfare maximizing quantity of money and this optimal quantity of money arises if and only if the willingness of agents to accept money happens to be just right. In summary, for money to be essential in our model, it is necessary that the store have a relatively low operating cost, and either barter must be relatively difficult or agents

\textsuperscript{4}Shi (2006, p. 644) defines money as “essential” if it “improves the efficiency of resource allocations relative to an economy without money.” He argues essentiality is a property to be sought in a model of money because (a) we want to understand how much money can improve welfare, and (b) a model with non-essential money is not likely representative because it would not likely stand the test of time.
must discount the future at a relatively high rate.

The organization of the paper is as follows. Section 2 reviews additional relevant literature. Section 3 presents our economy’s microfoundation, a store is introduced that can mediate trade when barter fails. In section 4, we extend the model by assuming the existing store considers the profitability of issuing IOU’s. We define an equilibrium for this money-store economy under the assumption that centralized mediation market is contestable, providing an existence theorem and a stability theorem. In section 5, we examine welfare issues, and we conclude with some discussion in section 6.

2 Related Literature

Our model is an extension of the Kiyotaki and Wright (1993) search theoretic model\(^5\), most closely related in form to the presentation of Ljungqvist and Sargent (2000, pp. 602-604). In this respect, our work is less related to that of Starr (2003) mentioned in the introduction, and more related to Ritter (1995), Williamson (1999), Cavalcanti et al (1999), and Berentsen (2006). Ritter (1995, p. 140) claims the KW model is incomplete because it does not explain where money comes from. He shows that if a coalition of KW agents is large enough, it will have the credibility and incentive to successfully issue fiat money\(^6\). Using a variant of Ritter’s model, Berentsen (2006) demonstrates the relevance of public knowledge, and the ability to punish the money issuer, for supporting the circulation of money when it is being issued by a private, revenue-maximizing monopolist. Williamson (1999) develops a model where banks issue bank notes to mitigate a mismatch between the receipt of investment payoffs and the

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\(^5\)Kiyotaki and Wright (1989) pioneered this approach to monetary theory, delineating the circumstances under which commodity money would endogenously arise to ameliorate the trading friction naturally present in the trading environment.

\(^6\)In more detail, Ritter (1995) first shows no individual KW agent has sufficient incentive to issue money, even though there would be individual benefit if the social contrivance could arise. However, when a coalition of agents can bond together, with the objective of maximizing the utility of the average member in the coalition, the incentive problem can be overcome. The coalition’s optimization problem recognizes the over issuance of money hurts the long run utility of money, whereas the individual optimization problem does not. The size of the coalition determines its credibility in issuing the fiat money and, as in the KW model, credibility must reach a threshold for money to circulate. Thus, the ability for fiat money to arise and circulate ultimately depends upon the coalition being large enough. The coalition need not be a government, but it is reasonably interpreted as such.
desired timing of consumption. Cavalcanti, et al. (1999) create a structure where banks are motivated to issue bank notes to obtain profit from float; the bank can consume when the note is issued, but does not have to pay for the consumption until the note is redeemed.

Shi (2006) identifies three ways researchers have introduced mediums of exchange that can compete with money in the KW framework: mechanism design, bilateral credit, and middlemen. Our store is tangentially related to each. Mechanism design involves creating a resource allocation mechanism, compatible with the incentives of the agents, as a form of public record keeping. Using this approach, Kocherlakota (1998) shows money can play an essential role only if the public record keeping device is imperfect. Bilateral credit involves giving an agent the ability to issue a nontransferable IOU to complete a trade when there is only a single coincidence of wants. Credit is an imperfect substitute for money because the creditor must monitor the debtor, and not consume, until the debtor is in a position to repay the IOU. This allows bilateral credit and money coexist in equilibrium, and Shi (1996) shows the introduction of bilateral credit enhances efficiency. Adding middlemen has involved either assuming middlemen can improve information about quality of the good being purchased by making an investment, or assuming middlemen improve the likelihood of the match by investing to acquire capacity to store more types of goods. With either of these approaches, money and trade through middlemen can coexist when the investment cost for middlemen is not too high.

When we allow our agents participate in both a decentralized and centralized market, we

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7 Cavalcanti, et al. (1999) model banks as being required to have deposit credits on hold with a clearing house that are debited when traders redeem the notes issued by the bank. They show that reserve requirement keeps banks from over-issuing the private money notes, which would cause a break down of the banking system.

8 Howitt (2005) presents yet another approach. He uses a creative modification of the Starr (2003) framework, where profit seeking firms create shops that reduce agent search costs by easily located. This approach can be viewed as providing a microfoundation for the trading post story of the type introduced by Shapley and Shubik (1977). Because Howitt’s shops have fixed costs of operation and can only mediate the trade of a limited number of goods, money can arise as an additional medium of exchange, for firms must accept money in exchange for goods in order to generate enough trading volume to cover the fixed costs.

9 This discussion of bilateral credit is based upon Shi (1996). See Li (2001) and Corbae and Ritter (2002) for other examples.

10 See Li (1999).

are adopting an approach initiated by Lagos and Wright (2005). While the microfoundational workings of the decentralized market are explained, those of the centralized market are not. This modeling choice is naturally open to the criticism that the model does not have a complete microfoundation. However, the tradeoff is greater parsimony\(^\text{12}\). The parsimony not only keeps our primary results from being obscured, but it also leaves open a variety of interesting extensions that would not otherwise be possible. We discuss possible extensions in our concluding remarks.

### 3 A Store Economy (Without Money)

Except for the inclusion of a store technology, the economy we consider is a discrete time version of the Kiyotaki and Wright’s (1993) model as described by Ljungqvist and Sargent (2000, pp. 602-604). Agents specialize in production and consumption. There is a continuum of infinitely lived agents and a continuum of goods, each normalized to one. The exogenous variable \(x\) denotes the proportion of goods providing utility to a given agent, and also equals the proportion of agents that can obtain utility from consuming any particular good. The utility \(u\) of consuming units of a consumable good is given by \(u(y) = y\)\(^\text{13}\). Production is a random draw from the continuum of goods, which yields one unit of a particular good to the producing agent. The produced good does not provide utility to the agent, but can be stored without cost. Production may only occur at the moment when one time period proceeds to the next. After production occurs, a new period begins with a social interaction modeled as a bilateral random matching process. The probability that one agent is matched with another is \(\theta\), where

\[
0 < \theta \leq 1.
\]

\(^{12}\)He, Huang, and Wright (2005, p. 639) similarly contend that introducing a centralized market is a useful simplification, one that allows them to more easily develop and examine a banking sector in the KW framework.

\(^{13}\)This utility function is beyond what is needed here, where there is no technology available for dividing goods, but it will useful once a store technology is introduced that allows the store to divide goods.
Because no good has any special characteristics, the probability that a good will be accepted in exchange is independent of the good held. Thus, $x$ is the probability that any given trader will want any given good in exchange (i.e., the probability of a single coincidence of wants), and $x^2$ is the probability an agent in a match will experience a double coincidence of wants. It is assumed

\[ 0 < x < 1, \]  

so a double coincidence of wants may or may not occur when two agents are matched. Each agent experiences the disutility $\varepsilon$ whenever a good is accepted in trade, and

\[ 0 < \varepsilon < 1. \]  

This positive transactions cost $\varepsilon$ rules out the formation of commodity money. Because trading for a non-consumable good would generate the cost without positively affecting future trading or consumption opportunities. Thus, an agent will only trade for a consumable good. Further, an agent will trade and consume if a double coincidence of wants occurs because the agent is allowed produce after consumption and enter the next period with a unit of good. Because no agent can divide a good, each agent trades one unit of good for one unit of good in a barter transaction and experiences the net utility

\[ u - \varepsilon = 1 - \varepsilon. \]  

Each agent discounts the future at the same constant rate. The discount factor $\beta$ denotes the current period value of one unit of utility received in the next period, with

\[ 0 < \beta < 1. \]  

Given these traditional barter economy assumptions, the value function for an agent looking forward from the beginning of any period, is

\[ V_G^B = \theta x^2 (1 - \varepsilon) + \beta V_G^B, \]
which implies each agent expects a lifetime discounted expected utility of

$$V^B_G = \frac{\theta x^2 (1 - \varepsilon)}{1 - \beta}. \quad (7)$$

Rather than introducing money to mediate exchange, we introduce a store technology. To keep the model as simple as possible, we assume the store technology is initially owned by a single agent, but the centralized mediation market provided by the store is perfectly contestable. In particular, we assume any agent can and will purchase the store technology at the same operating cost as the owner if the owner is receiving any net benefit from store ownership. This allows us to define and examine a contestable market equilibrium, and the model can be kept simple because there is no net advantage to the store owner in equilibrium, for all such advantages have been arbitrated away.

The store technology allows a clearing house to be constructed that mediates trade and allows the store to divide any good. Any good for good trade involves the store accepting one unit of good from an agent, while providing $\gamma$ units of some other good in exchange. The store must charge a markup on the goods it buys to cover its costs, and the ability to divide goods enables the store to shave off a fee for each transaction it completes. Assuming the fee for trading through the store is the same for each good and public information, agents would make their plans observing $\gamma$ in the range

$$0 < \gamma \leq 1 \quad (8)$$

The mediation capability of the store allows agents not completing a barter transaction to nonetheless trade, consume, and obtain utility in the period. Because of the store’s need to charge a markup, agents recognize trading through the store is a second best option. Thus, we assume agents wait to experience the social interaction and then seek to trade through a store if and only if the social interaction does not yield a barter transaction.

We assume the store can fully coordinate the trade of agents who do not barter. The rationale for this assumption is the following. When the $1 - \theta$ agents are not matched, it is not because they cannot be matched, but because they do not happen to meet another
agent. An advantage of a store is people know its location. If the \(1 - \theta\) agents not matched in the social interaction go to the store, the store can ensure each agent is matched with another. The store would also have the opportunity to coordinate trade for the \(\theta(1 - x^2)\) agents who are matched with another in the social interaction but do not experience a double coincidence of wants. Thus, there will be \(1 - \theta x^2\) agents who arrive at the store because they do not complete a barter transaction. Because production and matching each occur at random, the preferences of these \(1 - \theta x^2\) agents would be randomly distributed, just as all agents preferences were distributed before the social interaction. If these agents could have experienced a double coincidence of wants in the social interaction, but did not because the matching process is random rather than selective, then it is reasonable to think that the store could, by paying a cost to implement a selective matching process, coordinate the trade of any measure of agents.

It is assumed, trade through the store also generates the transactions cost \(\varepsilon\) for agents, just as in a barter trade. Consequently no agent would be willing to trade through the store if \(\gamma < \varepsilon\). Trading through the store in this case would produce a net current period loss of utility, while holding over good to the next period would not. Thus, the store must set the trading rate so

\[
\gamma \geq \varepsilon
\]  

(9)

Given the assumptions just described, the value function for an agent in the Store Economy is

\[
V^S_G = \theta x^2 (1 - \varepsilon + \beta V^S_G) + (1 - \theta x^2) (\gamma - \varepsilon + \beta V^S_G)
\]  

(10)

The first term on the right side of (10) is the value associated with the barter possibility, while the second term is the value associated residual trade opportunity. Solving for \(V^S_G\), each agent expects the lifetime discounted expected utility

\[
V^S_G = \theta x^2 (1 - \varepsilon) + (1 - \theta x^2) (\gamma - \varepsilon)
\]

(11)

Comparing (11) and (7), we see that the introduction of the store increases the expected lifetime utility of the agent if and only if \(\gamma > \varepsilon\). That is, we know the introduction of the
store is Pareto improving, if the store can attract customers.

The store captures as revenue the fraction $1 - \gamma$ of each unit of good delivered by an agent, and $1 - \theta x^2$ agents trade through the store. Therefore, measured in units of utility, the expected total revenue of the store is $(1 - \gamma) (1 - \theta x^2)$.

Assume the store operation cost is also measured in units of utility, and can be paid by the store using any good. We consider two cases. In one case, the store’s operating cost is the fixed cost

$$\bar{c} > 0.$$  \hfill (12)

and in the other case, the store’s operating cost is the variable per unit cost

$$\hat{c} > 0.$$  \hfill (13)

Using these cost assumptions, the store’s profit level is

$$\pi = (1 - \gamma) (1 - \theta x^2) - \bar{c}$$  \hfill (14)

in the fixed cost case and

$$\pi = (1 - \gamma - \hat{c}) (1 - \theta x^2)$$  \hfill (15)

in the variable cost case. A store is viable if and only if it earns a non-negative profit; i.e., $\pi \geq 0$.

The store is a profit maximizing entity. If a profit is earned it would be in the form of good that could be distributed back to the owner of the firm, which would enhance the well being. The contestable market assumption implies the existing store must prevent other stores from capturing the market. To do this, that the existing store must set its mark up so the store earns zero profit. Because the existing store will not earn a profit in equilibrium, there is no need to model how profits are distributed, which simplifies the model.

**Definition (Equilibrium for a store Economy):** Given conditions (1)-(3) and (5) on the barter economy structure, and conditions (12)-(13) on the store technology operation cost,
an Equilibrium for the Store Economy is a value for $\gamma$ such that (i) the store is viable; i.e., $\pi \geq 0$, (ii) the store attracts customers; i.e., $\gamma \geq \varepsilon$, (iii) no other store can enter and capture the mediation market from the existing store; i.e. there is no $\gamma' \geq \gamma$ yielding store profit $\pi' \geq 0$, and (iv) the store’s profit is maximized under conditions (i)-(iii).

**Theorem 1 (Existence of an Equilibrium for the Store Economy):** A Contestable Market Equilibrium for the Store Economy exists if and only if

\[
\tilde{c} \leq (1 - \theta x^2) (1 - \varepsilon) \quad \text{(fixed cost case)} \tag{16}
\]

\[
\tilde{c} \leq 1 - \varepsilon. \quad \text{(variable cost case)} \tag{17}
\]

Proof: See the Appendix

In either the fixed or variable cost case, the store’s desire to maximize its profit is superseded by its need to retain control of the contestable market. The trading rate $\gamma$ in the contestable market equilibrium is as attractive as it can be for agents because it must be set so profit equals zero to prevent entry. In the fixed cost case, the equilibrium trading rate is $\gamma = 1 - \frac{\varepsilon}{(1-\theta x^2)}$. This rate worsens as the double coincidence likelihood increases because the store has fewer customers over which to spread the fixed cost. Using this trading rate to compare life in the Store Economy to that in the Barter Economy, we find $V^S_G - V^B_G = \frac{(1-\theta x^2) (1-\varepsilon) - \varepsilon}{1-\beta}$, which implies a store will improve well-being when it can arise. In the variable cost case, the equilibrium trading rate $\gamma = 1 - \tilde{c}$ is independent of the volume of trade through store. Using this rate, we find $V^S_G - V^B_G = \frac{(1-\varepsilon) - (1-\theta x^2) \tilde{c}}{1-\beta}$. Again, for the variable cost case, we find the store will enhances well-being if it can arise\(^\text{14}\).

In both the fixed cost and variable cost cases, as the cost of store operation goes to zero, the equilibrium trading rate $\gamma$ goes to one. With zero store operation costs, the store in

\(^{14}\)Store viability implies $\tilde{c} \leq 1 - \varepsilon$ and we know $0 < 1 - \theta x^2 < 1$. Together these two conditions imply $1 - \varepsilon > (1 - \theta x^2) \tilde{c}$, so $V^S_G - V^B_G = \frac{(1-\varepsilon) - (1-\theta x^2) \tilde{c}}{1-\beta}$ implies $V^S_G > V^B_G$. 

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a contestable market provides a perfectly efficient medium of exchange, and the expected lifetime utility of an agent is $V^S_G = \frac{(1 - \epsilon)}{1 - \beta}$.

4 A Money-Store Economy

The existence theorem for the Store Economy indicates there is reason to think a store would naturally arise. The agent possessing the technology initially has an incentive to introduce it because economic profit could be earned out of equilibrium until the competition from the assumed contestability reduces profit to zero in equilibrium. This begs another question. If the store solves the double coincidence of wants problem in the Barter Economy, then is there any niche left for money? We will now show there is a remaining niche. By recognizing this niche, we explain why private money might naturally arise as a second medium of exchange. The money holder speculaets that money will provide a medium of exchange in the next period, rather than the store, which allows store’s markup to be avoided.

Consider the following innovation in the store’s technology. In addition to allowing agents to trade good for good, suppose the store allows an agent can also choose to accept an IOU from the store valued at $\gamma$ units of desired good. Because future consumption is discounted, there is no reason to hold the IOU unless the store offers some additional incentive. The incentive we explore is a store policy that allows the IOU to be traded to other agents, so the agent who redeems the IOU at the store need not be the agent who initially receives the IOU. Transferable modern day store gift cards are an example of such an IOU. When the IOU is held from one period to the next, it becomes money held as a store of value. When the IOU is traded from one agent to another, it becomes money circulating as a medium of exchange.

As is typical in search theoretic models of money, assume production cannot occur as long as money is held. Let $M$ denote the measure of agents holding money, so $1 - M$ is the measure not holding money. Figure 1 presents the trading possibilities for an agent in the Money-Store Economy, depending upon whether the agent enters the given period holding money or holding good.

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Figure 1: The Money-Store Economy
Let \( V_M \) and \( V_G \) denote the lifetime discounted expected utility of an agent who starts a period holding money and good, respectively. The value function for a money holder can be written as

\[
V_M = \theta (1 - M) x P (1 - \varepsilon + \beta V_G) + \left(1 - \theta (1 - M) x P\right) \max_p \{ p \beta V_M + (1 - p) (\gamma - \varepsilon + \beta V_G) \} \tag{18}
\]

The first term is associated with the possibility of trading money for good. The probability that an agent holding money is matched with an agent holding good is \( \theta (1 - M) \), and \( x \) is the probability that the agent holding money wants the good held by the other. Thus, the probability of a “money match” is \( \theta (1 - M) x \). When a money match occurs, the money holder moves to a chance node, where the agent holding good decides whether or not to accept the money. We assume all agents, including the money holder, subjectively perceive other agents will accept money with probability \( P \in [0, 1] \). If the money is accepted, the money holder trades money for good, consumes, produces, and enters the next time period holding good. This transaction occurs with probability \( \theta (1 - M) x P \) and yields \( 1 - \varepsilon + \beta V_G \) units of lifetime expected utility.

The second term of value function (18) is associated with a money holder who is not able to trade money to another agent for good. This disappointing outcome occurs with probability \( 1 - \theta (1 - M) x P \), and the agent holding money proceeds to a choice node, where a decision must be made as to whether to continue holding money or not. This choice involves solving the problem

\[
\max_p \{ p \beta V_M + (1 - p) (\gamma - \varepsilon + \beta V_G) \}, \tag{19}
\]

where \( p \) is the probability with which the agent chooses to hold money. When the agent chooses to hold money, consumption is forgone, no production occurs, and the next period is experienced as a money holder, which yields an expected lifetime utility of \( \beta V_M \). Alternatively, when the agent decides against holding money, \( \gamma \) units of good are received from the store, consumption occurs, production occurs, and the next period is experienced as a good
holder, which yields the expected lifetime utility of $\gamma - \varepsilon + \beta V_G$. When $\gamma - \varepsilon + \beta V_G < \beta V_M$, the pure money holding strategy $p = 1$ is best. When $\gamma - \varepsilon + \beta V_G > \beta V_M$, the pure strategy of not holding money $p = 0$ is best. When $\gamma - \varepsilon + \beta V_G = \beta V_M$, the agent is indifferent between holding money and not, so any mixed strategy $p = [0, 1]$ is as good as any other such strategy. Regardless of the solution of problem (19), the second term of the value function represents value an agent holding money expects when the agent is unable to trade money to another agent for good\textsuperscript{15}.

The value function for a good holder can be written as

$$V_G = \theta (1 - M) x^2 (1 - \varepsilon + \beta V_G) +$$

$$(1 - \theta (1 - M) x^2) \max_p \{\max \{p \beta V_M + (1 - p) (\gamma - \varepsilon + \beta V_G)\}, \beta V_G\}$$

(20)

The first term of this value function comes from the barter possibility. The probability of being matched with another good holder is $\theta (1 - M)$, and the probability of double coincidence of wants is $x^2$. Therefore, the probability that a good holder will experience a barter trade is $\theta (1 - M) x^2$. When barter occurs, each agent in the pair consumes, produces, and enters the next period starting with good. Thus, barter yields $1 - \varepsilon + \beta V_G$ units of utility. Because this is at least as much utility as any other possibility, a barter trade is executed whenever possible. Consequently, the expected value associated with the barter possibility is $\theta (1 - M) x^2 (1 - \varepsilon + \beta V_G)$.

The second term of value function (20) is associated with the possibility that the agent does not complete a barter transaction, which occurs with probability $1 - \theta (1 - M) x^2$. In this event, the agent must decide whether or not to hold over the good to the next period. Utility $\beta V_G$ is obtained from holding over the good, and $\max_p \{p \beta V_M + (1 - p) (\gamma - \varepsilon + \beta V_G)\}$ is obtained from solving problem (19) otherwise. To understand why problem (19) is always

\textsuperscript{15}In order for an agent to have ever decided to hold money $\gamma - \varepsilon + \beta V_G \leq \beta V_M$ would have to have held at some point in the past. Thus, in order to have this problem to solve, we know that the agent will not choose $p = 0$, and we could present the value function of the money holder as $V_M = \theta (1 - M) x P (1 - \varepsilon + \beta V_G) + (1 - \theta (1 - M) x P) \beta V_M$. We choose to present the more general value function for the money holder as we do in (18) so that the reader can see all of the possible trading options in the value function, even those that will not be used in equilibrium.
faced when good is not held over, first note that one reason for not completing a barter trade is the agent is matched with another good holder, but no double coincidence of wants occurs. In this case, the agent holding good proceeds to the store and solves problem (19) to determine whether to accept money from the store or trade good for good. Alternatively, barter might not occur because the good holder is matched with a money holder. If the money holder does not want the good held by the good holder, then the good holder again proceeds to the store and solves problem (19). If the money holder wants the good holder’s good, the good holder must decide whether to accept the money holder’s money, or not. This again involves solving problem (19) for the alternative is trading the good to the store in exchange for good, as long as holding over good is not better.

At this point, it is useful to illustrate why the introduction of money offers potential benefit. The probability that a good holder will trade away good for money is $p_M x$, for this requires that the good holder be matched with a money holder, that the money holder wants the good holder’s good, and that the good holder will accept the money. Because a good holder experiences barter with probability $\theta (1 - M) x^2$, the probability that an agent holding good neither experiences a barter match nor trades after a money match is $1 - \theta (1 - M) x^2 - p_M x$. In this case, the agent proceeds to the store. In the Store Economy, the probability that a good holder will arrive at the store is $1 - x^2$. Subtracting the former probability from the latter, we find the difference is $\theta M x(p - x)$, which is positive and increasing in $M$ when $p > x$. Thus, when the likelihood a good holder will accept money exceeds the likelihood of a single coincidence of wants, more money in circulation makes it less likely that good holders will have to go to the store and pay the store’s markup. This is why the circulation of money has the potential to increase well being.

Note that we are maintaining the assumption that agents cannot divide good. One implication of this indivisibility assumption is that the money holder receives all of the trading surplus, $1 - \gamma$, available when a money holder and good holder meet and trade. It is the possibility of obtaining this surplus, obtaining one unit of good through money trade in the future rather than $\gamma$ units through store trade in the present, which motivates an agent
to accept money. If goods were divisible, it would be reasonable to expect the trading surplus to be split between the two traders. In particular, rather than receiving one unit of good from the good holder, the money holder would receive something less, though more than γ, so the good holder would also reap a present benefit from the monetary trade. To allow for this divisibility, we would have to complicate the model with bargaining assumptions, or a price mechanism, so the split of the trading surplus is determined by the model. While we believe this extension offers the exciting possibility of modeling price determination in the search theoretic framework in a more natural way than has been done before, we have chosen not to pursue this here for it would blur the main result. However, what the reader should understand is that, if we can find conditions where the issuance and holding of money is incentive compatible in this context, where a producer gets nothing in the current period from trading for good for money, but is entirely motivated by the potential surplus gained in a future period, we should also be able to do so in the case where divisibility would allow money trade to provide immediate rewards. That is, the indivisibility assumption implies we are examining an extreme case, where we should have the most difficulty finding conditions under which money can arise and circulate.

We now turn to the store’s problem. For the store to exist, condition (9) must hold, and if γ ≥ ε then no good holder will ever hold over good from one period to the next, for nothing is gained. This implies all agents who arrive at the store holding good will either trade good for good or trade good for money. Let α denote the fraction that trade good for good, so 1 − α trade good for money. Let δ denote the fraction of those arriving at the store with money that trade money for good, so that 1 − δ choose to hold money. It follows that the store experiences the following transactions. There are $\alpha (1 - M) (1 - \theta (1 - M) x^2 - p\theta M \bar{x})$ agents who deliver good to the store in exchange for good. There are $(1 - \alpha) (1 - M) (1 - \theta (1 - M) x^2 - p\theta M \bar{x})$ agents who deliver good to the store in exchange for money. There are $\delta M (1 - \theta (1 - M) xP)$ agents who deliver money to the store in exchange for good. It follows that the revenue of
the store is

\[ R = (1 - \gamma)\alpha (1 - M) \left( 1 - \theta (1 - M) x^2 - p\theta Mx \right) + \\
(1 - \alpha) (1 - M) \left( 1 - \theta (1 - M) x^2 - p\theta Mx \right) - \gamma\delta M \left( 1 - \theta (1 - M) xP \right) \]  

(21)

To keep the model as simple as possible, we assume the store sets its trading rate with the expectation that a stationary equilibrium will prevail, where the new money it issues will equal to old money it redeems, so

\[ (1 - \alpha) (1 - M) \left( 1 - \theta (1 - M) x^2 - p\theta Mx \right) = \delta M \left( 1 - \theta (1 - M) xP \right) \]  

(22)

Condition (22) and revenue definition (21) together imply the revenue function for the store reduces to \( R = (1 - \gamma) (1 - M) \left( 1 - \theta (1 - M) x^2 - p\theta Mx \right) \).

Examining this condition, we see that, as in the store economy, the store’s revenue depends upon the markup \( 1 - \gamma \) and the number of agents holding good who end up trading through the store, which is \( (1 - M) (1 - \theta (1 - M) x^2 - p\theta Mx) \). The store’s equilibrium profit level is

\[
\pi = (1 - \gamma) (1 - M) \left( 1 - \theta (1 - M) x^2 - p\theta Mx \right) - \bar{c} \text{ for the fixed cost case and (23)}
\]
\[
\pi = (1 - \gamma - \bar{c}) (1 - M) \left( 1 - \theta (1 - M) x^2 - p\theta Mx \right) \text{ for the variable cost case.}
\]

We can now define an equilibrium for the Money-Store Economy:

**Definition (Equilibrium for Money-Store Economy):** Given conditions (1)-(4) and (12)-(13), an equilibrium for the Money-Store Economy is a quadruple \((p, P, \gamma, M)\) such that (i) agents maximize their lifetime expected utility; i.e., the value functions (18) and (20) are satisfied, (ii) each agent holds money if and only if it is optimal; i.e., \( p \) solves problem (19), (iii) agents have rational expectations; i.e., \( p = P \), (iv) the store is viable; i.e., \( \pi \geq 0 \), (v) the existing store prevents viable entry; i.e., there is no \( \gamma' > \gamma \) yielding store profit \( \pi' \geq 0 \), (vi) the store attracts customers; i.e., \( \gamma \geq \varepsilon \), (vii) the store maximizes profit \( \pi \) as given by condition (23), and given conditions (iv)-(vi), (viii) there is a determinant amount of
money in circulation; i.e., $0 < M < 1$, (ix) both the store and money mediate exchange; i.e., $\beta V_M = (\gamma - \varepsilon + \beta V_G)$.

Conditions (i)-(vii) are conditions on agent and store behavior. Condition (viii) indicates we are looking for an equilibrium where money circulates, and condition (ix) indicates we are looking for an equilibrium where money does not dominate the store as a medium of exchange, nor vice versa.

**Lemma 1** *(Excess Expected Value of Holding Money):* If $0 < M < 1$, and $\varepsilon \leq \gamma < 1$, then

$$\beta V_M - (\gamma - \varepsilon + \beta V_G) \geq 0 \iff P - x \leq \frac{\gamma - \varepsilon}{\beta \theta x (1 - M) (1 - \gamma)}.$$  

(24)

Proof: See Appendix

**Lemma 2** *(Equilibrium Quantity of Money):* In any equilibrium where $P \neq x$ and $\gamma \neq 1$,

$$M = \frac{(1 - \gamma) \beta \theta x (P - x) - (\gamma - \varepsilon)}{\beta \theta x (P - x) (1 - \gamma)} = 1 - \frac{(\gamma - \varepsilon)}{\beta \theta x (P - x) (1 - \gamma)}.$$  

(25)

Proof: From Lemma 1, the equilibrium condition $\beta V_M = (\gamma - \varepsilon + \beta V_G)$ implies $P - x = \frac{\gamma - \varepsilon}{\beta \theta x (1 - M) (1 - \gamma)}$. Solving for $M$ then yields condition (25).

**Lemma 3** *(Equilibrium Trading Rate Restriction):* In an equilibrium where $0 < M < 1$, the trading rate must satisfy

$$\varepsilon < \gamma < \frac{\beta \theta x (P - x) + \varepsilon}{1 + \beta \theta x (P - x)}.$$  

(26)

Proof: Using the solution (25) to replace the quantity of money in the equilibrium condition $0 < M < 1$, solving for $\gamma$ directly yields condition (26).

**Lemma 4** *(Restriction on willingness to accept money):* In an equilibrium where $0 < M < 1$, the probability of accepting money must satisfy

$$P > x + \frac{(\gamma - \varepsilon)}{\beta \theta x (1 - \gamma)}.$$  

(27)
Proof: If $P \leq x + \frac{(\gamma - \varepsilon)}{\beta \theta x (1 - \gamma)}$, then condition (25) indicates $M \leq 0$, which violates the money circulation condition $0 < M < 1$.

Lemma 4 indicates agents must have enough confidence in money in order for it to circulate in equilibrium. When this confidence exists, and money circulates, condition (25) allows us to understand how the store’s choice for $\gamma$ influences the quantity of money in circulation. As the store’s markup increases so that the trading rate $\gamma$ decreases toward the transactions cost $\varepsilon$, store trade becomes less attractive and money becomes the dominant medium of exchange; i.e., $M \to 1$. Alternatively, as the markup decreases and the trading $\gamma$ increases and approaches $\frac{\beta \theta x (P-x) + \varepsilon}{1 + \beta \theta x (P-x)}$, store trade becomes more attractive and money is driven out of the economy; i.e., $M \to 0$. The expression $\frac{\beta \theta x (P-x) + \varepsilon}{1 + \beta \theta x (P-x)}$ is increasing in $P$ and $\beta$. Thus, when there is more confidence in money, or when agents discount the future less, a higher trading rate $\gamma$ is necessary to drive money out of the economy.

In the Kiyotaki and Wright (1993) economy, $P > x$ must hold for money to circulate. Condition (27) indicates, in our Money-Store Economy, agents must generally have even more confidence in money for it to circulate. This is because monetary trade does not just compete with the barter opportunity, but it must also compete with the store. The store is no competition when its markup is so high that $\gamma = \varepsilon$. However, as the store’s trading rate increases above this threshold level, agents must have increasing confidence in money for it to circulate while competing with the store.

**Theorem 2 (Non-Existence of a Monetary Equilibrium for Fixed Cost Case):** When store operation cost is the fixed cost $\bar{c}$ and expectations are rational (i.e. $P = p$), no equilibrium exists with $\varepsilon \leq \gamma < 1$ for the Monetary Economy.

Proof: See Appendix

What we learn from Theorem 2 is, if a store of the fixed cost type can exist, it will drive money out of the economy. Increasing the trading rate reduces the store’s markup, which makes the store more attractive. This increases the number of traders through the store and reduces the number of money holders. Profits increase because the fixed cost is spread over
more traders, and the store becomes increasingly efficient. Eventually, the attractiveness of the store drives money out of the economy, so a monetary equilibrium cannot exist.

**Theorem 3 (Existence of a Monetary Equilibrium for Variable Cost Case):** When the variable store cost $\hat{c}$ satisfies

$$\frac{1 - \varepsilon}{1 + \beta x (1 - x)} < \hat{c} < 1 - \varepsilon$$  \hspace{1cm} (28)

then a set of equilibria exists for the Monetary Economy with a store trading rate

$$\gamma = 1 - \hat{c}.$$  \hspace{1cm} (29)

The set of equilibria contains an infinite number of elements, parameterized by the expectation parameter $P^*$, and $P^*$ satisfies

$$P = x + \frac{1 - \varepsilon - \hat{c}}{\beta x \hat{c}} < P^* \leq 1,$$  \hspace{1cm} (30)

and the equilibrium quantity of money satisfies

$$M^* = 1 - \frac{1 - \varepsilon - \hat{c}}{\beta x \hat{c} (P^* - x)}.$$  \hspace{1cm} (31)

Proof: See Appendix

From conditions (29) and (31), we see that, as the per unit store operation cost $\hat{c}$ increases to $1 - \varepsilon$, the equilibrium trading rate $\gamma$ decreases to $1 - \varepsilon$ and economy’s money supply increases to 1. That is, money becomes a more dominant medium of exchange as the cost of operating the store increases. As the store operation cost $\hat{c}$ decreases to $\frac{1 - \varepsilon}{1 + \beta x (1 - x)}$, the expectation $P^*$ must increase to 1 for a monetary equilibrium to exist, the trading rate increases to $\frac{\beta x (1 - x) + \varepsilon}{1 + \beta x (1 - x)}$ and the money in circulation decreases to zero.

The threshold level of confidence for money to exist is $\hat{P} = x + \frac{1 - \varepsilon - \hat{c}}{\beta x \hat{c}}$ in condition (30), so there are multiple monetary equilibria associated with expectations in the range $P^* \in (\hat{P}, 1]$. As the store operation cost $\hat{c}$ increases, the expectation $P^*$ can be lower with money still
circulating, and the lower confidence bound is reached as \( \hat{c} \) reaches \( 1 - \varepsilon \). As \( x \) increases, agents must have more confidence in money for it to exist, and there is some value for \( x \) large enough that no store can exist and support the circulation of money, because barter becomes so effective. Condition (30) indicates that if people discount the future enough (\( \beta \) small enough), if the store operations cost is low enough (\( \hat{c} \) small enough), if a match in the social interaction is too unlikely (\( \theta \) small enough), or a single coincidence of wants too unlikely (\( x \) small enough), then a monetary equilibrium cannot be supported even if agents are entirely confident that money will be accepted (i.e., \( P = 1 \)).

In a monetary equilibrium, when an agent arrives at the store with money, it is equivalent to arriving with good because in either case the store offers \( \gamma \) units of good in exchange. In equilibrium, all such agents are indifferent between holding the money into the next good and trading with store and consuming this period. Because of this indifference in equilibrium, our model does not distinguish how much money is being redeemed at the store, how much money the store is issuing, nor how long a particular agent holds money. Imposing the indifference assumption on our equilibrium has allowed us to keep our model simple, so as to keep the focus on how this framework can be used to endogenize the introduction of private money.

**Theorem 4 (Stability of Monetary Equilibrium—Variable Cost Case):** Let \( M^* \) denote the equilibrium quantity of money, when \( \beta V_M = (\gamma - \varepsilon + \beta V_G) \). For all \( M \in [0, M^*) \), \( \beta V_M > (\gamma - \varepsilon + \beta V_G) \), and for all \( M \in (M^*, 1) \), \( \beta V_M < (\gamma - \varepsilon + \beta V_G) \).

Proof: See Appendix

Theorem 4 is intuitive. When there is no money in circulation, or very little, accepting money offers the largest advantage it can offer, for the probability of being matched with a good holder who will accept the money in exchange is as high as it can be. Condition (28) ensures each agent views holding money as being better than holding good in this situation, so good holders would exchange good for money at any opportunity, and the quantity of money would increases. When the quantity of \( M \) is closer to its equilibrium value, there are
fewer good holders for a money holder to be matched with, so the expected value associated with holding money is closer to that associated with not holding money. Condition (28) ensures that there can also be so much money that its expected value is less than that associated with holding good. In this situation, money holders would exchange money for good at any opportunity, and the quantity of money would decrease toward the equilibrium quantity. Thus, the existence of the store as a competing medium of exchange prevents private money being overissued.

Theorem 4 also indicates that a store equilibrium, as defined in section 5, cannot persist when the store operation cost satisfies condition (28) and there is enough confidence in money to satisfy condition (30). To see this, assume \( \gamma = 1 - \hat{\gamma} \), so that the existing store in the store economy equilibrium is earning zero profits, and all agents not making barter transactions are trading through the store. Theorem 4 indicates every agent would prefer to go to a store with trading rate \( \gamma = 1 - \hat{\gamma} \) that also issues money, for higher expected value could be obtained by accepting money from such a store. Such a store would not only rightly perceive that it could capture the mediation market because of the added expected value offered to agents, but it would also rightly perceive that pure profit would be earned on each new unit of money issued in the disequilibrium as money is initially introduced. Thus, to prevent the entry of such a store in the contestable market, the existing store would be motivated to create the money and earn the profit in disequilibrium by issuing it.

5 Welfare

To examine welfare, we use the criterion \( W = MV_M + (1 - M) V_G \). In equilibrium, when \( \beta V_M = (\gamma - \varepsilon + \beta V_G) \), the value functions (18) and (20) reduce to \( V_M = \frac{(1-M)\theta P(1-\gamma)}{1-\beta} \) and \( V_G = \frac{(1-M)\theta x^2(1-\gamma)+(\gamma-\varepsilon)}{1-\beta} \), so we can write the welfare function as

\[
W = \frac{M (1-M) \theta P x (1-\gamma)}{1-\beta} + \frac{(1-M) ((1-M) \theta x^2 (1-\gamma) + (\gamma - \varepsilon))}{1-\beta} \tag{32}
\]

Note that, if a monetary economy cannot form so \( M = 0 \), the welfare function reduces to \( W = V_G \), where the welfare level \( V_G \) in condition (32) is the same as that for the Store
Economy in condition (11). Because money holding precludes production and reduces the likelihood of barter, we see in condition (32) that the portion of welfare associated with good holding is monotonically decreasing in $M$. The portion of condition (32) associated with money holding indicates that more money contributes to welfare when the quantity of money is low, but detracts from welfare when the quantity of money is high. An increase in the money level increases the fraction of agents who reap the reward of trading money for good, but more money reduces production and reduces the probability of a money match. The former effect dominates when the money level is low, but the latter effect dominates when the money level is high.

The effect of the money level on welfare is given by the derivative

$$\frac{dW}{dM} = \frac{\theta x (1 - \gamma) (P - 2x - 2(P - x)M)}{1 - \beta} - \frac{(\gamma - \varepsilon)}{1 - \beta} \tag{33}$$

Evaluating at $M = 0$, with $\gamma = 1 - \hat{c}$, we find that the introduction of money to the Store Economy can increase welfare only if

$$P > \bar{P} = 2x + \frac{1 - \hat{c} - \varepsilon}{\theta x \hat{c}} \tag{34}$$

When $P = 1$, as in the monetary equilibrium for the Kiyotaki and Wright (1993) fiat money economy, and $\hat{c} = 1 - \varepsilon$ is imposed to rule out the existence of the store, then condition (34) reduces to $x < \frac{1}{2}$, which is the condition Ljungqvist and Sargent (2000) show must hold in order for fiat money to be welfare improving.

For the Store-Money Economy, when money endogenously arises and finds its equilibrium value, the equilibrium quantity of money will be welfare maximizing only by chance. The optimal quantity of money, found by setting the derivative (33) equal to zero, can be written as

$$\hat{M} = \frac{P - 2x}{2(P - x)} - \frac{(1 - \hat{c} - \varepsilon)}{2\theta x \hat{c} (P - x)}$$

Comparing this optimal quantity of money to the equilibrium quantity of money $M^*$
given by (31), we find

\[ M^* \leq \hat{M} \iff P^* \leq \hat{P} = \frac{(1 - \hat{c} - \varepsilon)(2 - \beta)}{\beta \theta x \hat{c}} \]

That is, we find that, when an interior optimum exists for the quantity of money, there exists a unique confidence level \( \hat{P} \) that will bring forth the optimal quantity. Full confidence in fiat money (i.e., \( P = 1 \)), or any confidence level \( P^* > \hat{P} \), brings forth more money than is optimal, while \( P^* < \hat{P} \) brings forth too little. The following theorem delineates how the introduction of money affects welfare, in general, depending on the store's operation cost \( \hat{c} \) and likelihood \( x \) of a single coincidence of wants.

**Theorem 5 (Welfare):** For any monetary equilibrium, when \( x < \frac{1-\beta}{2-\beta} \), the effect of an increase in the equilibrium quantity of money affects welfare depends upon the store's operation cost:

- **Case 1:**
  \[ \frac{(1-\varepsilon)}{(1+\beta \theta x(1-x))} < \hat{c} < \frac{(2-\beta)(1-\varepsilon)}{(2-\beta)+\beta \theta x} \iff \frac{\delta W}{\delta M^*} > 0 \text{ for all } P^* \in [\hat{P}, 1] \]

- **Case 2:**
  \[ \frac{(2-\beta)(1-\varepsilon)}{(2-\beta)+\beta \theta x} < \hat{c} < \frac{1}{(1-\beta)+\beta \theta x^2} \iff \frac{\delta W}{\delta M^*} > 0 \text{ for all } P^* \in [\hat{P}, \hat{P}] \text{ and } \frac{\delta W}{\delta M^*} < 0 \text{ for all } P^* \in (\hat{P}, \hat{P}] \]

- **Case 3:**
  \[ \frac{(1-\varepsilon)}{1+\beta \theta x(1-x)} < \hat{c} < (1-\varepsilon) \iff \frac{\delta W}{\delta M^*} < 0 \text{ for all } P^* \in [\hat{P}, 1] \]

However, when \( x \geq \frac{1-\beta}{2-\beta} \), an increase in the quantity of money only reduces welfare in that \( \frac{(1-\varepsilon)}{1+\beta \theta x(1-x)} < \hat{c} < (1-\varepsilon) \iff \frac{\delta W}{\delta M^*} < 0 \text{ for all } P^* \in [\hat{P}, 1] \).

Proof: See Appendix

Theorem 5 indicates that, in order for the introduction of money to be welfare enhancing, agents must discount the future enough and barter cannot be too easy. When barter is difficult enough or when agents discount the future enough, the level of the store's operation cost determines whether or not more money enhances welfare. When the store has a low operation cost (i.e., Case 1), more money only increases welfare, and more money is present in the economy as the confidence agents have in money increases from \( \hat{P} \) to 1. When the store

\[ ^{16} \text{Note that in case 1 and case 2 welfare increases with equilibrium quantity of money and not quantity of money.} \]
operation cost is high (i.e., Case 3), more confidence in money only decreases welfare. The intermediate case (Case 2), where there is an interior optimum for the quantity of money, occurs when the store cost is in an intermediate range. In this case, regarding the impact on welfare, agents can have either too much confidence in money (i.e., \( P^* > \hat{P} \)) or too little (i.e., \( P^* < \hat{P} \)).

6 Discussion

We have presented a model in which fiat money naturally arises from the competition among profit seeking stores for a contestable, centralized mediation market. Holding money does increase the likelihood of trade in the decentralized market, but an agent will only hold money from one period to the next because it offers the possibility of reducing the transactions costs of trade. Thus, the essentiality of money in our model arises primarily from money’s ability to reduce transactions costs, not simply from its ability to resolve the double coincidence of wants problem.

The equilibrium quantity of fiat money in our model is determined by the microfoundation of the model, not by government. As in traditional search theoretic models of fiat money, the confidence agents have in money is important for determining whether the circulation of fiat money can be supported. However, in our model, for a given confidence level, the equilibrium quantity of money is stable, meaning there is one to one mapping from money confidence to the quantity of money in circulation.

This has implications for monetary policy. For example, suppose government introduces a form of fiat money before our store recognizes it can profit by introducing money. Assume agents view government money as equivalent to that introduced by the store in our model. Our model indicates government could introduce a quantity of money up to the amount supported as the equilibrium quantity for our model, and doing so would crowd out the ability of the store to introduce money. However, government could not get agents to voluntarily accept an amount of money above this because agents would prefer to trade their good to the store in exchange for good. In general, our model suggests that recognizing competing
mediums of exchange is important for evaluating the scope of money’s impact.

Our model also has fiscal policy implications. Specifically, taxes on trade can impact the quantity of money in circulation in our model and, consequently, the level of welfare. Only by coincidence will the level of confidence in money support an equilibrium quantity of money that maximizes welfare. We can examine the implementation of a transaction tax by noting in our model this would effectively increase the transactions cost. Condition (31) indicates this would increase the equilibrium quantity of money, providing an increase in welfare if the original equilibrium quantity of money was too low. From the discussion in the previous paragraph, we know a government attempt to increase the money supply would not accomplish this same improvement in welfare. This suggests further exploration of models with competing mediums of exchange may enhance the understanding of how welfare may be affected by the interaction of monetary and fiscal policies.

The framework we have used here is amenable to a number of extensions. The model could conceivably be given a complete microfoundation, so both money and its competing intermediary arise from the same microfoundation. The dynamics of the model could be specified, so we could watch the economy evolve from the point at which money is initially introduced, rather than just examining the steady state. A model with determinate prices might be obtained by relaxing the assumption that agents cannot divide goods. Money and banking might fruitfully be examined by explicitly introducing banks, and allowing loans to be made with fractional reserves.

Finally, when it comes to explaining why a medium of exchange can compete with other mediums by providing value to traders, our model emphasizes both the importance of the ability to commit and transactions cost. When an agent trades good for good through the store, it is the store’s ability to honor the good for good contract that makes the store attractive to the agent. However, to provide this ability to commit, the store must charge agents a transactions cost. By using money as the medium of exchange, agents can avoid this transactions cost, but must face potential commitment uncertainty. Other agents may not accept the money. In our model, more of this commitment uncertainty leads to less
money in circulation. Further work may allow us to more fully understand how the ability
to commit and transactions costs interact as they explain why observe particular trade
mediation forms\textsuperscript{17}.

7 Appendix: Proofs

Proof of Theorem 1: Because profits are continuously decreasing in $\gamma$, any level of $\gamma$ that
generates a positive profit level cannot be sustained, for a viable competing store can be
created with $\gamma' > \gamma$, so all agents will prefer the competing store. Thus, to prevent the
successful entry of a competing store, the existing store must adjust $\gamma$ to eliminate profit.
In the fixed cost case, the store earns zero profit when $(1 - \theta x^2) (1 - \gamma) - \bar{c}$, or when $\gamma = 1 - \frac{\bar{c}}{1 - \theta x^2}$. Given this value for $\gamma$, the store attracts customers if and only if $\gamma \geq \varepsilon$, which implies $\bar{c} \leq (1 - \theta x^2)(1 - \varepsilon)$. In the variable cost case, the store earns zero profit when $\gamma = 1 - \hat{c}$. Given this value for $\gamma$, the store attracts customers if and only if $\gamma \geq \varepsilon$, which implies $\hat{c} \leq 1 - \varepsilon$.

Proof of Lemma 1: Let $p^*$ denote the optimal value for $p$ in problem (19). If $\gamma \geq \varepsilon$, then the condition $\underset{p}{\text{Max}} \{ \underset{p}{\text{Max}} \{ p \beta V_M + (1 - p) (\gamma - \varepsilon + \beta V_G) \}, \beta V_G \}$ in value function (20) becomes $p^* \beta V_M + (1 - p^*) (\gamma - \varepsilon + \beta V_G)$, for no agent will choose to hold good from one period to the next. Using the value functions (18) and (20) to construct the difference $D = \beta V_M - (\gamma - \varepsilon + \beta V_G)$, we can write

$$D = \beta \theta x (1 - M) (P - x) [(1 - \varepsilon + \beta V_G) - (p^* \beta V_M + (1 - p^*) (\gamma - \varepsilon + \beta V_G))] - (\gamma - \varepsilon).$$

Letting $A$ denote the quantity $\beta \theta x (1 - M) (P - x)$, the last condition can be rewritten as $D = A [1 - \gamma - p^* D] - (\gamma - \varepsilon)$, which implies $(1 + p^* A) D = A [1 - \gamma] - (\gamma - \varepsilon)$. Because $0 < \beta \theta x (1 - M) < 1$ and $-1 < P - x < 1$, we know $-1 < A < 1$. Knowing $A > -1$ and $0 \leq p^* \leq 1$, we know $(1 + p^* A) > 0$. Thus, $(1 + p^* A) D = A [1 - \gamma] - (\gamma - \varepsilon)$ implies the sign of $D$ depends upon the sign of $A [1 - \gamma] - (\gamma - \varepsilon)$. Condition (24) follows directly.

Proof of Theorem 2: By Lemma 2, the quantity of money in equilibrium must satisfy condition (25). Using this quantity of money, the profit function for the store in the fixed

\textsuperscript{17}We thank Gabrielle Camera for recognizing this interaction between commitment and transactions costs
cost case becomes

\[ \pi = \frac{(\gamma - \varepsilon)}{\beta \theta x (P - x)} \left[ (1 - p\theta x) + \frac{(\gamma - \varepsilon)(p - x)}{\beta (1 - \gamma) (P - x)} \right] - \tilde{c} \]

From Lemma 2, we know \( P - x > 0 \) must hold for an equilibrium to exist with \( \varepsilon \leq \gamma < 1 \). Under rational expectations \( P - x > 0 \) must also hold. It then follows that the store’s profit is strictly increasing in the trading rate \( \gamma \). Any store would therefore be motivated to increase \( \gamma \) to a level such that \( \gamma > \frac{\beta \theta x (P - x) + \varepsilon}{1 + \beta \theta x (P - x)} \), violating condition (26) of Lemma 3. The contestable market will not prevent this increase in the trading rate, but rather encourage it. This is sufficient to complete the proof.

**Proof of Theorem 3:** Together, the dynamic program conditions (18) and (20) and the equilibrium condition \( \beta V_M = (\gamma - \varepsilon + \beta V_G) \) imply that the quantity of money in equilibrium must satisfy condition (25). Using this quantity of money, the profit function for the store in the variable cost case becomes

\[ \pi = (1 - \gamma - \tilde{c}) \left[ \frac{(\gamma - \varepsilon)}{\beta \theta x (1 - \gamma) (P - x)} \left[ (1 - p\theta x) + \frac{(\gamma - \varepsilon)(p - x)}{\beta (1 - \gamma) (P - x)} \right] \right] \]

Profit per store customer, \( (1 - \gamma - \tilde{c}) \), is decreasing in \( \gamma \). Because \( P > x \) and \( P = p \) by the rational expectations assumption, the number of store customers, \( \frac{(\gamma - \varepsilon)}{\beta \theta x (1 - \gamma) (P - x)} \left[ (1 - p\theta x) + \frac{(\gamma - \varepsilon)(p - x)}{\beta (1 - \gamma) (P - x)} \right] \), is increasing in trading rate \( \gamma \). By setting \( \gamma = \varepsilon \), the store obtains a high profit per unit, but has no customers. As \( \gamma \) increases from \( \varepsilon \) to \( 1 - \tilde{c} \), the number of customers continuously increases, while the profit per unit decreases continuously to zero. Therefore, a profit maximum exists in this domain for \( \gamma \). Contestability implies the trading rate must be set at \( \gamma = 1 - \tilde{c} \) to eliminate the profit that would entice competing stores to enter. With \( \gamma = 1 - \tilde{c} \), condition (25) implies the money in circulation is the amount presented as condition (31). Because \( \beta V_M = (\gamma - \varepsilon + \beta V_G) \), agents are indifferent between holding money and trading through the store, so agents are indifferent about the value of \( p \). Thus, with \( \gamma = 1 - \tilde{c} \), any value for the expectation \( P \) that supports the circulation of a positive quantity of money can qualify as an equilibrium expectation. The restriction (28) ensures that there exists at least one such value for \( P \). Condition (30) gives the range of values for \( P \) that support a money
supply \( M \) such that \( 0 < M < 1 \), and condition (31) gives the equilibrium money supply. Because all equilibrium conditions are satisfied for values of \( P \) satisfying condition (30), the proof is complete.

Proof of Theorem 4: Let \( Z = \frac{(1-c-x)}{\beta \theta x (1-M)c} \). Because \( c < 1 - \varepsilon \) by assumption (28), \( Z \) is monotonically increasing in \( M \) for all \( M \in [0,1] \). Because \( \gamma = 1-c \) and \( \beta V_M = (\gamma - \varepsilon + \beta V_G) \) in equilibrium, condition (24) implies \( P - x = Z^* \), where \( Z^* = \frac{(1-c-x)}{\beta \theta x (1-M^*)c} \). Because \( Z \) is increasing in \( M \), it follows that \( P < x + Z \), for all \( M \in (M^*, 1) \), and condition (24) implies \( \beta V_M < (\gamma - \varepsilon + \beta V_G) \). Because \( Z \) is increasing in \( M \), it also follows that \( P > x + Z \), for all \( M \in [0, M^*) \), and condition (24) implies \( \beta V_M > (\gamma - \varepsilon + \beta V_G) \).

Proof of Theorem 5: From (31), we know the equilibrium quantity of money \( M^* \) is strictly increases from zero to a quantity we will call \( M_{\text{max}}^* \) as \( P \) increases over the interval \( P \in [\bar{P}, 1] \). The second derivative of the welfare function (32) is \( \frac{\partial^2 W}{\partial M^2} = -\frac{2 \theta c \varepsilon (P-x)}{1-\beta} \), which is strictly negative for all \( M \in [0,1] \) when \( P > x \). Since \( P > x \) in any monetary equilibrium, this negative second derivative implies we have three possible cases. As \( P \) increases over the interval \([\bar{P}, 1]\), so that \( M^* \) increases over the interval \([0, M_{\text{max}}^*]\), welfare either strictly increases (Case 1), strictly increases to a peak and then strictly decreases (Case 2), or strictly decreases (Case 3).

From (36), we know Case 1 occurs when \( \bar{P} > 1 \) because \( P < \bar{P} \) must hold for all \( P \in [\bar{P}, 1] \), which implies \( M^* < \hat{M} \) for all \( M^* \in [0, M_{\text{max}}^*] \). Using the definition of \( \bar{P} \) in (36), \( \bar{P} > 1 \) implies \( \hat{c} < \frac{(2-\beta)(1-\varepsilon)}{(1+\beta \theta x (1-x))} \). In order for \( M^* \) to be an equilibrium value, (28) implies \( \hat{c} > \frac{(1-\varepsilon)}{(1+\beta \theta x (1-x))} \). These last two inequalities hold simultaneously if and only if \( x < \frac{1-\beta}{2-\beta} \). Thus, when \( x < \frac{1-\beta}{2-\beta} \), Case 1 holds if and only if \( \frac{(1-\varepsilon)}{(1+\beta \theta x (1-x))} < \hat{c} < \frac{(2-\beta)(1-\varepsilon)}{((2-\beta)+\beta \theta x)} \).

From (36), we know Case 2 occurs when \( P < \bar{P} < 1 \). This is because \( P < \bar{P} \) must hold for all \( P \in [\bar{P}, 1] \), which implies \( M^* < \hat{M} \) for all \( M^* \in [0, \hat{M}] \), but then \( P > \bar{P} \) must hold for all \( P \in [\bar{P}, \bar{P}] \), which implies \( M^* > \hat{M} \) for all \( M \in (\hat{M}, M_{\text{max}}^*) \). Using the definitions of \( \bar{P} \) and \( \bar{P} \) in (30) and (36), \( P < \bar{P} \) implies \( \hat{c} < \frac{(1-\varepsilon)(1-\varepsilon)}{(1+\beta \theta x (1-x))} \). Using the definition of \( \bar{P} \) in (36), \( \bar{P} < 1 \) implies \( \hat{c} < \frac{(2-\beta)(1-\varepsilon)}{(1+\beta \theta x (1-x))} \). In order for \( M^* \) to be an equilibrium value, (28) implies \( \hat{c} > \frac{(1-\varepsilon)}{(1+\beta \theta x (1-x))} \). These last two inequalities hold simultaneously if and only if \( x < \frac{1-\beta}{2-\beta} \).
Thus, when \( x < \frac{1-\beta}{2-\beta} \), Case 2 holds if and only if 
\[
\frac{(2-\beta)(1-\theta)(1-c)}{(2-\beta)+(\theta x^2)} < \hat{c} < \frac{(1-\beta)(1-c)}{(2-\beta)+(\theta x^2)}.
\]

From (36), we know Case 3 occurs when \( \hat{P} < \hat{P} \) because \( P > \hat{P} \) must hold for all 
\( P \in [\hat{P}, 1] \), which implies \( M^* > \hat{M} \) for all \( M^* \in [0, M_{\text{max}}] \). Using the definitions of \( \hat{P} \) and 
\( \hat{P} \) in (30) and (36), \( \hat{P} < \hat{P} \) implies \( \hat{c} > \frac{(1-\beta)(1-c)}{(2-\beta)+(\theta x^2)} \). The quantity 
\( \frac{(1-\beta)(1-c)}{(2-\beta)+(\theta x^2)} \) is strictly less than \( (1-\varepsilon) \) for all the admissible parameter values \( 0 < \beta < 1 \), \( 0 < x < 1 \), \( 0 < \varepsilon < 1 \), and 
\( 0 < \theta \leq 1 \). Thus, when \( x < \frac{1-\beta}{2-\beta} \), Case 3 holds if and only if 
\( \frac{(1-\beta)(1-c)}{(2-\beta)+(\theta x^2)} < \hat{c} < (1-\varepsilon) \).

When \( x = \frac{1-\beta}{2-\beta} \), 
\[
\frac{(1-\varepsilon)(1-c)}{(2-\beta)+(\theta x^2)} = \frac{(1-\beta)(1-c)}{(2-\beta)+(\theta x^2)},
\]
so it is not possible to find a value for \( \hat{c} \) such that satisfies either Case 1 or Case 2. However, we may still find a value for \( \hat{c} \) such that 
\( \frac{(1-\beta)(1-c)}{(2-\beta)+(\theta x^2)} < \hat{c} < (1-\varepsilon) \), so Case 3 may still hold when \( x = \frac{1-\beta}{2-\beta} \). When \( x > \frac{1-\beta}{2-\beta} \), 
\[
\frac{(2-\beta)(1-\varepsilon)}{(2-\beta)+(\theta x^2)} < \frac{(1-\varepsilon)}{(1-\beta)+(\theta x^2)} \quad \text{and} \quad \frac{(1-\beta)(1-c)}{(2-\beta)+(\theta x^2)} = \frac{(2-\beta)(1-\varepsilon)}{(2-\beta)+(\theta x^2)},
\]
Thus, it is not possible to find a value for \( \hat{c} \) such that satisfies either Case 1 or Case 2. However, we may still find a value for \( \hat{c} \) such that 
\( \frac{(1-\beta)(1-c)}{(1-\beta)+(\theta x^2)} < \hat{c} < (1-\varepsilon) \), so Case 3 may still hold when \( x > \frac{1-\beta}{2-\beta} \). Thus, we can conclude 
that when \( x \geq \frac{1-\beta}{2-\beta} \), only Case 3 may hold, which is sufficient to complete the proof.

References

Review 50 (7); 1683-1698.

4; 289-323

with Money. Journal of Money, Credit and Banking, 40(6); 1295-1308.

management in a random matching model. Journal of Political Economy 107 (5); 929- 
945.

public record keeping, Economic Theory 24; 933–51.


