Setting Safety Stock Using a Fill Rate

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In class, we have spent most of our time talking about having a certain percentage chance of not running out, which we have called a "cycle service level" (CSL). Many people in industry talk about a "fill rate," which is a different way to measure service. In this handout, we explore the difference between the two measurements and see how to use a Fill Rate.

1 Fill rate vs. Cycle Service Level

A fill rate is the percentage of demand that is met on time. The cycle service rate is the probability that we do not run out while waiting for an order to come in. In the long run, this should be the percentage of order periods where we don't run out.

If we have $CSL = 0.9$, there is a 90% chance we don't run out while waiting for an order to come in. During the year, suppose we order 10 times. On average, we won't run out during 9 of those 10 order cycles. During 1 order cycle per year, we run out. How much do we run out by? We don't know, but if we carry enough safety stock that we only run out once every 10 order cycles, we have quite a bit of safety stock, so it's unlikely that we would run out by very much. The result is that we would have very few unhappy customers. So $CSL = 0.9$ means carrying a lot of safety stock and having very few upset customers.

By contrast, if we set a fill rate of $FR = 0.9$, that means we are planning on a target of making 10% of our customers mad at us. If that does not sound like very good customer service to you, you are correct. $FR = 0.9$ would mean not carrying very much safety stock, and having poor customer service.

2 Formula

If our goal is to meet a particular FR, can use the following formula:

$$G_u(z) = \frac{Q}{\sigma_L}(1 - FR) \tag{1}$$

where Q is the order quantity, and σ_L is the standard deviation of demand over the lead time. $G_u(z)$ is the expected value of the number of units of shortage per order cycle. The formula comes from p. 268 of "Inventory Management and Production Planning and Scheduling," by Edward A. Silver, David F. Pyke, and Rein Peterson, 1998, henceforth known as SPP. Mathematically, the definition of $G_u(z)$ is:

$$G_u(z) = \int_{D=k}^{\infty} (D - z) f(D) Dz \tag{2}$$

where $f(D)$ is the standard normal probability density function (pdf). If you tell me z, evaluating z is not hard (SPP, p. 735):

$$G_u(z) = f(z) - Pr\,(D \geq z) * z \tag{3}$$

where $f(z)$ is the probability of z in the standard normal distribution. In Excel, this is easy to compute:

$$G_u(z) = NORMDIST(z, 0, 1, FALSE) - [1 - NORMSDIST(z)] * z. \tag{4}$$

So it is easy to find out what $G_u(z)$ is, for different value of z. But we want to know what z will give us the value of $G_u(z)$ we want. From SPP,p. 736, there is a formula:

$$z = \frac{a_0 + a_1 k + a_2 k^2 + a_3 k^3}{b_0 + b_1 k + b_2 k^2 + b_3 k^3 + b_4 k^4},$$

where

$$k = \sqrt{ln\left(\frac{25}{G_u(z)^2}\right)} \tag{5}$$

$$a_0 = -5.3925569 \tag{6}$$

$$a_1 = 5.6211054 \tag{7}$$

$$a_2 = -3.8836830 \tag{8}$$

$$a_3 = 1.0897299 \tag{9}$$

$$b_0 = 1 \tag{10}$$

$$b_1 = -7.2496485 * 10^{-1} \tag{11}$$

$$b_2 = 5.07326622 * 10^{-1} \tag{12}$$

$$b_3 = 6.69136868 * 10^{-2} \tag{13}$$

$$b_4 = -3.29129114 * 10^{-3} \tag{14}$$

If you're thinking that's an ugly looking formula, you are right. Fortunately, I have entered the formula into Excel in a spreadsheet you can download from the class webpage.

3 Example

Suppose that our order size $Q = 500$, the standard deviation over the LT $\sigma_{LT} = 40$, and we want a fill rate of $FR = 0.9$. Putting these values into equation (1), we obtain:

$$G_u(z) = \frac{500}{40}(1 - 0.9) = 1.25.$$

Putting these values into the spreadsheet, we get a value of $z = -1.193$. If we define safety stock (SS) as $\sigma_{LT} * z$, in this case, we clearly have negative safety stock:

$$SS_{FR} = -1.19 * 40 = -47.7 \approx -48.$$

With a negative amount of safety stock, we are PLANNING to run out every order cycle. That doesn't seem to make any sense. What is essentially happening is this: Before we place an order, the fill rate is 100%. If we want the fill rate for the year to average out to be 90%, it has to be less than 90% during the reorder period. Actually, as it turns out, considerably less than 90%. Exactly how much less depends on how many orders we place per year, etc., and is taken into account in the formula.

In contrast, if we set $CSL = 0.9$, we get $z = 1.28$.

$$SS_{CSL} = 1.28 * 40 = 51.26 \approx 51.$$

Thus, having a target of $CSL = 0.9$ means carrying $51 - (-48) = 99$ more units of safety stock than having a target of $FR = 0$ would require.

4 Conclusions

Using a fill rate target of x does not require as much safety stock as would be needed to have a cycle service level of that same x. This doesn't mean one measurement is necessarily bad or the other is automatically good. Fill Rate is a very widely used measurement in industry, and although the formula is not something you would want to evaluate by hand, it's easy in Excel.